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A SYNTHESIS OF FUZZY RULE-BASED SYSTEM VERIFICATION

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Abstract

The verification of fuzzy rule bases for anomalies has received increasing attention these last few years. Many different approaches have been suggested and many are still under investigation. In this paper, we give a synthesis of methods proposed in literature that try to extend the verification of classical rule bases to the case of fuzzy knowledge modeling, without needing a set of representative input. Within this area of fuzzy V&V we identify two dual lines of thought respectively leading to what is identified as static and dynamic anomaly detection methods. Static anomaly detection essentially tries to use similarity, affinity or matching measures to identify anomalies within a fuzzy rule base. It is assumed that the detection methods can be the same as those used in a non-fuzzy environment, except that the formerly mentioned measures indicate the degree of matching of two fuzzy expressions. Dynamic anomaly detection starts from the basic idea that any anomaly within a knowledge representation formalism, i.c. fuzzy if-then rules, can be identified by performing a dynamic analysis of the knowledge system, even without providing special input to the system. By imposing a constraint on the results of inference for an anomaly not to occur, one creates definitions of the anomalies that can only be verified if the inference process, and thereby the fuzzy inference operator is involved in the analysis. The major outcome of the confrontation between both approaches is that their results, stated in terms of necessary and/or sufficient conditions for anomaly detection within a particular situation, are difficult to reconcile. The duality between approaches seems to have translated into a duality in results. This article addresses precisely this issue by presenting a theoretical framework which enables us to effectively evaluate the results of both static and dynamic verification theories.

Keywords

Rule based fuzzy systems, static verification approach, dynamic verification approach

1. Introduction

The importance of assuring the reliability of knowledge based systems (KBS) need not be emphasized. In that sense it should not come as a surprise that verification of the knowledge base (KB) within a KBS is object of extensive research. Until recently, most of the results have been achieved in the field of classical knowledge based systems [7], [9], [10], [12]-[14], mainly because V&V has received little attention in systems based on non-classical formalisms. Renewed interest in the modeling power of Lotfi Zadeh's fuzzy set theory [29]-[30] and the possibility it provides in reasoning with vague concepts seem to alter this. During the last few years, much of the research attention seems to have shifted towards the V&V-issue relating to fuzzy rule based systems.

In this paper, we identify two dual lines of thought one comes across when reviewing the literature in search for methods to tackle the problem of verifying a fuzzy rule base. These two main classes will be identified as static and dynamic verification approaches. The motivation of the confrontation between both branches in literature relating to the verification of fuzzy rule bases performed here, stems from the fact that the results of the respective approaches in terms of necessary and/or sufficient conditions to identify anomalies, are difficult to reconcile. The duality identified in approach seems to have translated into a duality in results. The analysis presented in this document is restricted to methods proposed in literature that try to extend the verification of classical rule bases to the case of fuzzy knowledge modeling, without needing sets of representative input to identify anomalies. Although one might wish to test a system on all possible inputs, in most systems this simply is not feasible. The class of papers that rely on the choice of a representative set of test cases is not included in this synthesis. Thereby we a priori exclude research in the field of neural network training and verification from consideration.

The paper is organized as follows. In section 2, the feasibility of verification by anomaly detection is addressed in a fuzzy rule based context. Section 3 introduces two dual lines of thought, respectively a static and a dynamic one, as to anomaly detection for fuzzy rule bases. In section 4, a zone of potential conflict will first be exemplified and consequently identified by means of a framework called the duality scheme. The principles underlying this framework will enable us in a next phase to explain the duality in outcome between both anomaly detection approaches identified in section 3. In section 5, the discussion in previous sections will be synthesized and some side remarks will be formulated. Section 6 outlines some issues of future research, while section 7 finalizes the discussion by summarizing the main issues dealt with in this paper.

2. Verification by Performing Anomaly Detection

Fuzzy set theory constitutes a superset of classical binary set theory [25]-[30]. The basic element is the notion of fuzzy set. This introduces a form of continuous logic, for now we have to be able to handle membership values in the interval [0,1]. Within a context of rule based expert systems it should be possible to use both classical and fuzzy sets in the knowledge modeling phase. This not only requires the new modeling formalism to still be able to handle classical sets, but also means that inference results in the case of crisp input into the fuzzy system should be in accordance with results obtained from a classical system. The requirement stated above has a direct implication on the construction of the inference engine : it should make use of what Dubois & Prade [11] called an implication-based rule design[1], because this guarantees compliance with the truth table of the classical implication. From a verification point of view, this has some interesting consequences. In designing a fuzzy rule based system that for any crisp input reproduces the same results as a classical system, one guarantees that erroneous inference results that appear out of the classical system persist when the same inputs are offered to the fuzzy system. In classical verification theory, research has succeeded in attributing errors that spring from the inference process, after certain input has been subjected to the system, to a set of *anomalies* within the constructed knowledge base. These anomalies are identified as inconsistency (i.e. incoherence), redundancy, circularity and deficiency. It should be clear that an anomaly is not an error. Errors spring from the inference process. Anomalies are but *symptoms* within the knowledge base of a knowledge system that point out the fact that the inference process could produce errors. The concept of anomaly can in fact be

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[1] The other main class of rules, conjunction based rules, is used mainly in fuzzy control. Cases of fuzzy control are not addressed in this paper. We restrict our attention to verification of fuzzy rule based expert system in which the fuzzy rule implication scheme complies with classical causality-based reasoning via the classical implication function.

connected to the knowledge base at a *conceptual* level, independent of any knowledge coding formalism. However, because knowledge based systems do not work at a conceptual level, but are designed in a specific knowledge representation formalism, both syntax and semantics of anomalies have to be (re)defined in terms of syntax and semantics of the knowledge representation language used to express the KB, i.c. fuzzy set theory. To be able to verify a knowledge model for anomalies, one has to discover the *manifestation* of the anomaly within the context of the chosen knowledge representation formalism. What matters in this context is the insight that, whether one uses a classical or a fuzzy formalism to code the knowledge, the underlying conceptual model remains unchanged. The set of anomalies identified out of research conducted in the context of classical rule based systems remains both relevant and exhaustive in a fuzzy rule based environment. Even though the kind of anomalies is unaltered, the manifestation of the anomalies is not. Verification research has to focus upon discovering the specific manifestations of these anomalies within the used knowledge encoding scheme.

3. Identification of Static versus Dynamic Anomaly Detection

In this phase of the discussion, we introduce two main lines of thought distinguished in fuzzy rule base verification literature. Identification is realized by means of denoting the main idea, the motives and the insights underlying each approach. identified

3.1 Verification as a Static Process

3.1.1 Motives, Goals and Basic Insights

In conceiving a verification theory for anomaly detection in fuzzy rule bases, one has to try to strive for an intuitively appealing approach. This commitment to intuition lies perfectly along the lines of thought of Zadeh's fuzzy set theory. The fact that fuzzy set theory is in fact merely a generalization of classical or crisp set theory allowing for a system to reason with vague concepts, opens further perspectives in a fuzzy verification context. It seems not unfeasible to try to transpose the major findings in the area of classical rule base verification to a fuzzy context. There exists a wide on the job experience with anomaly detection in rule bases of classical rule based systems. It mostly concerns knowledge systems that are modeled in first-order logic or, when no variables or functions are needed, in propositional logic. In addition, lots of tools surfaced on the market that were especially designed for performing anomaly detection in the latter type of context.

However, reuse of classical results or tools might be not that straightforward a task. By using fuzzy sets to represent knowledge, one gives rise to the possibility of partial equality between sets, an issue that has been covered by several papers in literature[20]-[24] and is illustrated in figure 1. In fuzzy systems, partial resemblance between sets is allowed, whereas in the context of classical systems a comparison between sets always either leads to an exact match or to a no-match. This implies that in a classical context a person is either tall or small, but never both, if we suppose these two labels define a partition of the length range. However, by considering 'tall' and 'small' as fuzzy labels, describing a fuzzy variable 'length', the outcome of a comparison in terms of the resemblance of sets now depends completely on the positions of their set-support[2], as can be seen in figure 1. A person measuring 1m70 is now both tall and small, be it to a different degree, indicated by the membership value of this specific height value within the considered fuzzy sets.

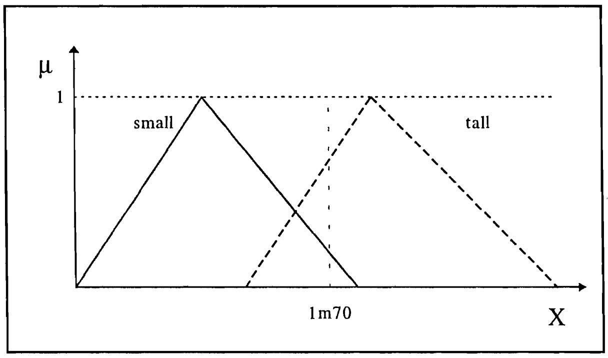

Figure 1 : person measuring 1m70, both small and tall

The relevance of this fundamental observation stems from the fact that about all classical formal anomaly definitions rely on the concept of equality between sets or on some very similar concept, like an 'is part of'-relationship. With this in mind, one has obtained a potential key to conceive a fuzzy rule base verification theory out of classical V&V results : classical formal anomaly definitions can simply be transposed to their fuzzy equivalents, by introducing a good fuzzy equivalence concept. The ultimate goal consists of transposing what is generally

[2] The support of a fuzzy set is defined as $S(A) = \{x \in X / \mu_A(x) > 0\}$, with μ the membership value of x to set A.

considered to be the strength of approach of classical anomaly detection in verifying crisp rule based systems : independent verification of the knowledge base and the inference engine. In a rule base environment solely consisting of classical sets, anomalies are detected by examining the syntax of the KB, although they are first identified semantically. Anomaly detection is performed on the KB of a KBS. The properties of the inference engine are assumed but not verified. The wish to transpose this major accomplishment to a fuzzy context explains why this branch of fuzzy verification literature is pinpointed as the static approach.

3.1.2 Core Concept

The lever that enables the transposition of results from a classical verification context towards a fuzzy context, is the ubiquitous presence of the concept of *equivalence* of sets in classical formal anomaly descriptions. The discovery of a *good fuzzy counterpart* to the concept of crisp equivalence of sets would enable the knowledge engineer to simply duplicate the anomaly detection methods from the crisp environment, with the slight adaptation of having to 'fuzzify' the concept of equivalence. Static anomaly detection essentially tries to use similarity, affinity or matching measures to identify anomalies within a fuzzy rule base. It is assumed that the detection methods can be the same as those used in a non-fuzzy environment, except that the formerly mentioned measures indicate the degree of matching of two fuzzy expressions.

Examples, or at least traces of this type of approach in fuzzy rule base verification literature can be found in Leung & So [5], Vanthienen & Wets [9], Scarpelli, Pedrycz & Gomide [8], Turksen & Wang [4] and Kinkielélé & Ayel [3].

3.2 Verification as a Dynamic Process

3.2.1 Motives, Goals and Basic Insights

A well founded theory of verification is a condition sine qua non for guaranteeing reliable functioning of a fuzzy rule based system. Any verification theory, however, has to earn itself a place within the modeling formalism underlying the built knowledge system. In this case, it concerns fuzzy set theory that knows how to handle fuzzy concepts, but in itself should not be fuzzy. This pleads for a verification theory that should be well embedded within the theoretical foundations of fuzzy set theoretic constructs. The introduction of fuzzy set theory in the modeling phase of a rule based knowledge system, not only implies the introduction of fuzzy sets. The step from classical sets towards fuzzy sets also requires the use of an adapted reasoning scheme, i.e. some sort of fuzzy implication function. The implication function, a basic element in knowledge modeling, therefore should not a priori be overlooked in conceiving a verification theory for any fuzzy rule based environment.

3.2.2 Core Concept

A dynamic anomaly detection method explicitly starts from the idea that anomalies are symptoms within the KB of a KBS pointing to potential erroneous output of the inference mechanism. However, anomalies are not errors. Errors stem from inference, when feeding specific input to the system. There remains however a relationship between errors in inference and anomalies in a KB, as explained in section 2. Identification of erroneous inference results therefore provides an excellent means of defining anomalies formally : *by imposing some type of constraint on the result of inference, that guarantees that the error does not occur, the possibility is offered to reason backwards and discover conditions to which the static knowledge base has to comply in order not to produce these errors.* By imposing a constraint on the results of inference for an anomaly not to occur, one creates definitions of the anomalies, that can, at least in a first phase, only be verified if the inference process, and thereby the fuzzy inference operator is involved in the analysis. This states that anomaly detection *always* has to pass via the inference process, the dynamics of the system, to eventually (as a final goal in verification research), if possible, come to static demands in terms of necessary and/or sufficient conditions which need to be imposed on the knowledge base in order not to manifest a specific anomaly. This is why this type of reasoning is identified as the dynamic approach to verification.

The main proponents of a dynamic verification approach are Yager & Larsen [1], Dubois, Prade & Ughetto [2]. Yager & Larsen were the first to introduce this type of verification in a fuzzy rule base. Their method of 'reflecting on the input' [1] allows to test a rule base for consistency. This in essence describes some sort of backward inferencing mechanism, that allows to translate the demand for normality, imposed on the fuzzy relationship that results from inference when one wishes it to be coherent, into an analysis of the input sets to the rule base. Dubois, Prade & Ughetto [2] then use the method of reflection on the input to try to obtain necessary and/or sufficient conditions for several scenarios within the rule base. The latter authors also conceive a dynamic redundancy analysis. This again follows the same line of thought : impose a suitable constraint on the results of inference for an anomaly not to occur and thereby create a suitable definition of the anomaly. In a second phase try to use this dynamic definition to obtain a static equivalent definition for analyzing the knowledge base, without explicitly taking into consideration the inference process. The suitability of the constraint is always determined by imposing compliance of the obtained results with the classical case. This guarantees that classical verification is only *extended* to include fuzzy cases (cf. section 2).

4. Identification of a Conflict

The findings in the previous part of the discussion, have lead to the description of a framework, i.e. the duality scheme (cf. infra), that positions both dynamic and static line of thought in relation to the evolution of verification in classical rule bases. The framework presented in this section of the text, immediately points out a potential zone of conflict between the static and the dynamic approach described in section 3. We claim that it is precisely this potential conflict that manifests itself when comparing most of the results of both approaches in fuzzy verification literature.

In a first phase, we construct this theoretical framework. In a second phase, some of the published results will be compared, just to illustrate the difficulty that is identified in reconciling the current status of both approaches towards verification of fuzzy rule based systems. The comparison merely underlines the relevancy of the duality scheme, presented in the next section, in explaining the observation of a duality in results established in literature.

4.1 The Duality Scheme

The framework presented in this section positions dynamic and static anomaly detection methods in relation to the evolution of verification through anomaly detection in classical rule bases. The duality scheme is represented in figure 2.

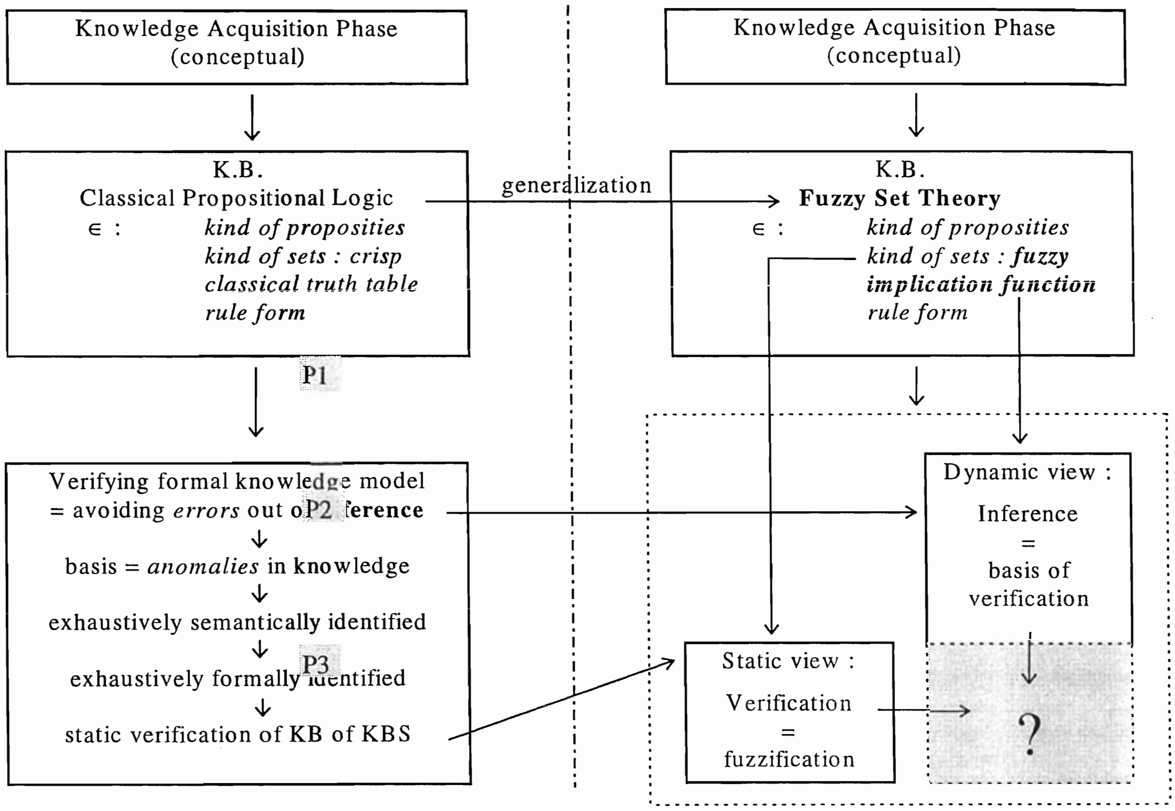

Figure 2 : the duality scheme

The framework in figure 2 is constructed as follows :

The left half of the figure, i.e. left of the vertical dotted line, represents the classical zone. This side encloses all the major realizations in the field of verification of classical propositional rule bases. These major realizations can be summarized by three principles[3] :

- Principle P1 : *Verification is done in function of the syntax and semantics of the specific knowledge representation formalism.*

This principle explains why the verification step can not directly follow the knowledge acquisition phase. Only after the conceptual knowledge model has been coded can one consider verification.

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[3] cf. Preece & Shingal [7] and Meseguer & Preece [10]

- Principle P2 : *Verification is done in order to avoid errors out of inference. The means to prevent those errors from occurring is found in the detection of their symptoms in the KB : anomalies.*

In relation with the previous principle, this latter one implies that one has to explicitly take into account the way of inferring results in order to obtain valid anomaly definitions. Principle P2 lies at the origin of the fact that it is always possible, by means of a dynamic analysis of the knowledge system, to impose a constraint on the results of inference in order to assure that a specific anomaly does not occur in the knowledge system, i.e. the dynamic verification approach. Yager & Larsen [1] illustrate this for a vast number of rule based logic encoding schemes, under which simple propositional logic and fuzzy logic, by using their method of reflecting on the input to detect any inconsistency in a rule based KB. They hereby create definitions of the anomaly, that is then verified by involving the inference process, and thereby the fuzzy inference operator in the analysis, even without having to feed a representative set of inputs to the system.

- Principle P3 : *Anomaly detection is performed on the KB of the KBS. Certain properties of the inference engine are assumed but not verified any more.*

This last principle is generally considered to be the strength of the *classical* anomaly detection approach. It allows for independent verification of inference engine and knowledge base. However, it remains necessary to specify those aspects of the inference mechanism upon which the results of this static kind of approach rely, whereas explicit testing of these inference engine properties is left behind. Verification research has succeeded in specifying anomalies in terms of the equivalence of the classical sets occurring in the rule base, or in terms of some related concept, e.g. the relationship 'is part of'.

When turning to the right hand side of the vertically dotted line in the duality framework, we enter the zone where fuzzy set theory makes its appearance in knowledge modeling. The introduction of fuzzy set theory engenders two major changes in the construction of a formal knowledge model : on the one hand there is the novelty of fuzzy sets, thus a new set formalism, on the other hand a adapted reasoning mechanism is introduced in the form of a fuzzy implication function.

The zone in the lower right half of figure 2, represented by means of the dotted frame, constitutes the relevant range in the context of fuzzy rule base verification via the technique of anomaly detection. From the discussion in the previous part of this paper and from the insight in the principles governing the left part of the duality framework of figure 2, it should now be clear where the duality in verification literature essentially finds its origin : dynamic anomaly detection methods are directly inspired upon principle P2, whilst the static counterparts of fuzzy verification literature try to directly transpose the acquirements underlying principle P3 to a fuzzy context.

4.2 Root of the Potential Conflict

The above analysis would be of little value if it did not make a point. It is a fact that the current realizations of the dynamic and the static anomaly detection methods are at least often difficult compatible with one another. This points in the direction of a potential conflict between both lines of thought. The power of the above described duality framework now enables us to put the observed difficulty to reconcile results in the right perspective. The potential zone of conflict within the above discussed duality scheme, is indicated by the light-gray zone in the lower right corner of figure 2. The origin of an in literature identified conflict between results that stem from a dynamic anomaly analysis and those that emerge from a static point of view on verification in fuzzy rule bases, in most cases relies on the fact that principle P2 and principle P3 can never be realized separately, because they can be but the respective deliverables of two consecutive steps in one and the same *sequential* verification research project. *This basic insight will in fact provide us not only with an explanation of why results in verification literature seem to differ according to the line of thought a verification theory belongs to, it also foresees in a means to normatively judge any proposed verification theory initiative.*

4.3 Compatible Motives, Incompatible Realisations

In the light of the in the previous section stated insight, both approaches of section 3 can be evaluated. With as main and direct motive the realization of principle P3, the static anomaly detection methods identified in fuzzy verification literature try to transpose the static anomaly detection methods from a non-fuzzy or classical environment into a fuzzy rule base environment by *fuzzifying* the concept of equivalence of sets. Static anomaly detection essentially tries to use similarity, affinity or matching measures to identify anomalies within a fuzzy rule base. It is assumed that the static detection methods can be the same as those encountered in a non-fuzzy environment, except that the formerly mentioned measures indicate the degree of matching of two fuzzy expressions. In that way these verification theory initiatives de facto uncouple verification and inference. By doing so, the probability of violating the major idea underlying principle P2, in that the specific inference mechanism cannot be omitted from any verification analysis, is not unthinkable. Taking principle P2 as a starting point in conceiving a verification theory for a fuzzy rule base environment, i.e. the idea behind the dynamic line of thought, causes no problems at all in that respect. It's even one of the main objectives of a P2-verification-analysis to be able to in the end realize principle P3, and obtain a static checking procedure in terms of necessary and/or sufficient conditions for verifying the KB of a rule based system. Unfortunately this is not yet the case, even though some major contributions have already been made by Dubois, Prade & Ughetto [2].

Where has the analysis along the lines of the duality scheme brought us at this point in the discussion ? The analysis essentially enables us to state that the foundations underlying both views on verification proposed in literature are not incompatible with one another. The incompatibility lies completely within the realizations of the motives governing both approaches. Principles P2 and P3 are not a priori inherently incompatible starting points in conceiving verification theories. Moreover, this is proven by the status in classical verification research, where results comply with both principles. So, why should this a priori be implausible in a context of fuzzy rule base verification ? In figure 2 this would imply that the frame of the static view shifts to right under the frame of the dynamic view, so that the parallel with the left hand side, the classical part, is complete. However up till now this has not happened yet. Why ? This is mainly due to the fact that static anomaly detection methods in general make abstraction of the semantics of the rules and thus leave the implication operator which is used out of the analysis. The abstraction of the type of reasoning scheme, i.e. the implication function, being a basic knowledge modeling element, directly conflicts with the idea behind principle P2. Therefore, we can but reject any static anomaly detection method that succumbs to this last pitfall.

4.4 Indicative Example of a Duality in Published Verification Strategies

In this section, we illustrate the assertion made in this document by comparing some of the outcomes of two recent publications, one conceived from a static point of view, the other one relying upon a dynamic verification of the fuzzy rule base. The analysis is based upon a specific result took from the paper by Leung and So [5] : the case of conflicting rule pairs[4]. The results obtained from this source document will be confronted with results obtained by Dubois, Prade & Ughetto [2] for the same kind of anomaly.

4.4.1 Description of Dual Approaches

The specific rule model that will be analyzed, consists of two rules of the following form

R1 : IF U is A1(x) THEN V is B1(y)
R2 : IF U is A2(x) THEN V is B2(y)

where A1 and A2, respectively B1 and B2 are fuzzy labels describing fuzzy variables U and V. U and V are defined on respective one-dimensional universes of discourse X and Y. We further assume that both rules are modeled as implication-based rules, cf. [2], [11]. This is completely consistent with the discussion in section 2.

Let's start with the definition of a conflicting rule pair {R1,R2}, governed by the previous model definition, as deduced by Leung and So. These authors start from the definition of a conflicting rule pair in the classical case, in order to, in a next step, come to a fuzzy case equivalent. Thus, assuming A1, A2, B1 and B2 are all crisp sets, they state that {R1,R2} is a conflicting rule pair if

$$A1 = A2 \ \ \& \ \ B1 \neq B2^{5}$$

[4] In order to keep things simple we make abstraction of the use of certainty factors, an element that is all but crucial in the verification strategy developed by Leung and So.

[5] i.e. the classical, and globally accepted definition of a conflicting rule pair

In a next step this definition is 'fuzzified' to handle fuzzy sets by introducing an *affinity measure A*, which replaces the equality of classical sets to come to the statement that fuzzy rule set {R1,R2} is contradictory or conflicting, if

$$A(A1,A2) \geq 0.5 \;\; \& \;\; A(B1,B2) < 0.5$$

The Affinity measure introduced by Leung and So is defined as

$$A(A1,A2) = M(A1 \wedge A2 \mid A1 \vee A2)^6$$

where $M(A1 \mid A2)$ is a similarity measure calculated by the following algorithm

IF	$N(A1 \mid A2) \geq 0.5$	**(1)**
THEN	$M(A1 \mid A2) = P(A1 \mid A2)$	**(2)**
ELSE	$M(A1 \mid A2) = (N(A1 \mid A2) + 0.5) * P(A1 \mid A2)$	**(3)**

where * denotes multiplication, $P(A1 \mid A2)=\max(\min(\mu_{A1},\mu_{A2}))^7$, $N(A1 \mid A2)=1-P(A1 \mid A2)$. $A(A1,A2)$ measures the similarity of $A1 \wedge A2$ given $A1 \vee A2$. That is, to what extent does the whole part of both A1 and A2 match the shared part of them. The affinity measure has following properties

i. If A1 and A2 are identical then $A(A1 \mid A2) = 1$
ii. $A(A1 \mid A2)$ is commutative
iii. $A(A1 \mid A2) = \min(M(A1 \mid A2),M(A2 \mid A1))$

A number of essential assumptions underlying this definition are

i. The sets used to model R1 and R2, even as all input sets are *convex* and *normalized.*
ii. The *implication function* does not play a role in the verification procedure.

In a second phase we confront the previous result with what Dubois, Prade & Ughetto [2] would state about the conflicting rule pair K={R1,R2}. For the set of implication-based rules K

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[6] $\mu_{A1 \wedge A2}=\min(\mu_{A1},\mu_{A2})$ and $\mu_{A1 \vee A2}=\max(\mu_{A1},\mu_{A2})$

[7] Do not be mislead by the notation, $P(A1 \mid A2)$ is a *commutative* expression in A1 and A2. Therefore it can be equivalently written as P(A1,A2).

to be *inconsistent*, assuming all fuzzy sets involved are normalized, following statement has to be fulfilled[8]

$$\exists\, x \in X : \mathrm{Sup}_y \min_{i=1\ldots2}(\mu_{Ai}(x) \rightarrow \mu_{Bi}(y)) < 1$$

i.e. there exists input data that together with K makes an inconsistent knowledge base, since the corresponding inferred[9] possibility distribution is not normalized.

A number of essential assumptions underlying this definition are

i. The fuzzy sets used are all convex and *normalized.*
ii. The *implication function* plays a determining role in coming to the concrete form of the above mentioned statement.

To illustrate the last assumption, we state some results deduced from Dubois, Prade & Ughetto [2], for two types of implication based rules, certainty and gradual rules

Proposition 1 : *necessary & sufficient incoherence cond. for 2 parallel certainty rules*

A knowledge base $K = \{ Ai(x) \rightarrow Bi(y) \mid i = 1..2\}$, governed by the above mentioned model, is incoherent if and only if,

1. $P(A1,A2) > 0$
2. $P(B1,B2) < 1$

For gradual rules, proposition 1 states but necessary incoherence conditions. Sufficient conditions for incoherence are stated in proposition 2.

Proposition 2 : *sufficient incoherence cond. for 2 parallel gradual rules*

A knowledge base $K = \{ Ai(x) \rightarrow Bi(y) \mid i = 1..2\}$, governed by the above mentioned model, is incoherent if

$$P(A1,A2) > P(B1,B2)$$

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[8] This statement is deduced from the reflection-on-the-input method by Yager & Larsen [1].

[9] using a Generalized Modus Ponens reasoning scheme.

It is now possible to compare both results covering this same situation. It should be clear from the fact that $(N(A1 \mid A2) + 0.5)$ always underlies the unity in expression **(3)**, that for the case of the above described model and the above described affinity criterion by Leung & So it is always true that

i. $P(A1,A2) \geq M(A1 \mid A2)$ & $P(A1,A2) \geq M(A2 \mid A1)$
ii. $P(B1,B2) \geq M(B1 \mid B2)$ & $P(B1,B2) \geq M(B2 \mid B1)$

Because of property iii. of the affinity measure A, these last two statements imply that

$$P(A1,A2) \geq A(A1,A2) \quad \& \quad P(B1,B2) \geq A(B1,B2) \qquad \textbf{(4)}$$

The possibility measure is therefore said to be an optimistic indication of the equivalence between two fuzzy expressions. This last result provides a direct means of comparing the assertions of Leung & So [5], a static approach toward fuzzy verification, with these deduced from the theory of Dubois, Prade & Ughetto [2], a dynamic approach towards fuzzy verification, as both results are now related to the same possibility measures. The outcome of the comparison is stated by means of the next four points.

i. It is clear that results are not identical. This implies that another {consistency, inconsistency}-classification of all possible scenario's is made by each group of authors.
ii. Whereas the results of Dubois, Prade & Ughetto make a clear distinction of results according to the implication function used, this is not the case with Leung and So.
iii. The boundary value of 0.5 looks at least empirical, whereas the numerical boundary values deduced by Dubois, Prade & Ughetto all stem from a clear demand for normality of fuzzy sets.
iv. Even if we simply replace the P-measure by the proposed A-measure in both propositions 1 and 2, in order to avoid iii., we still ca not resolve i.

It is now clear that, indeed, both verification theories have difficult to reconcile results. In the next subsection we analyze both results in more detail in order to come to an evaluation of the discrepancy in results.

4.4.2 Analysis of the Results

A common element in both verification theories describing the in previous section introduced fuzzy rule model {R1,R2}, is the assumption of normality of both fuzzy modeling and input sets. Why both verification theories make this assumption relies on the fact that all verification theories have to conform with a very simple idea : a single rule can never be incoherent. Testing this is quite straightforward : submit following rule set to the inconsistency test of the conceived verification theory to see if rule R1 is coherent in itself

R1 : IF U is A1(x) THEN V is B1(y)
$R_{redundant}$: IF U is X THEN V is Y (which is always true)

It can be verified quite easily that both approaches stated above only comply with this requirement when the fuzzy modeling sets used are normalized.

The big difference between both approaches relies in the fact that the dynamic view developed by Dubois, Prade & Ughetto [2] extend this normality assumption to the fuzzy output of the process of global inference. In that way one automatically obtains a condition which can be used to formulate the appropriate coherence condition. The static approach by Leung & So fails to see the link towards the condition of normality imposed on the fuzzy outcome of the process of global inference, although it imposes this same normalization constraint on the definition of all modeling and input sets. Instead the latter authors come up with a, from a theoretical point of view, rather arbitrary static definition of the anomaly known as an incoherent rule pair, making use of an affinity measure. What goes wrong ? The answer lies in section 2 of this document. The use of detecting any anomaly is based upon the fact that anomalies form the *symptoms*, within the constructed rule based model, of potential errors in the inference process. *Anomalies are not errors. Errors spring from the inference process.* Therefore the inference process should not be neglected in any verification analysis. *Things go wrong in the Leung & So-analysis at the moment that the authors fail to include the link to the inference error of which the studied anomaly is a symptom when constructing a static description of the anomaly in question.* This type of error in the reasoning is inherently avoided in a dynamic line of thought toward anomaly detection.

5. Conflict Resolution in Fuzzy Verification Literature : a synthesis

In this final part of the paper, we make a number of side remarks and draw the necessary conclusions out of the in previous parts of this discussion identified and exemplified duality in fuzzy verification literature. The identification of a conflict is one thing, to draw the right conclusions another. In that sense this last section is supposed to be a synthesis of an evaluation of fuzzy verification literature in which methods are proposed that try to extend the verification of classical rule bases to the case of fuzzy knowledge modeling, without needing any set of representative input.

5.1 Classical Anomalies Persist

All the evaluated verification methods are conceived from the point of view of anomaly detection. From the discussion in section 2 it should be clear that indeed this is still a permissible verification approach when migrating from a classical to a fuzzy rule based environment. However, to be able to verify a formal knowledge model for anomalies, one has to discover the *manifestation* of the anomaly within the context of the precise knowledge representation formalism, i.c. fuzzy set theory. It should be realized that, even though the kind of anomalies is unaltered, the manifestation of the anomalies is not.

5.2 Formal Anomaly Description

The previous analysis leaves no doubt about the fact that verification is tailored to fit into a specific knowledge coding formalism. This is synthesized as principle P1 in section 4.1. The fact that anomalies are identified as symptoms in the KB that point to the possibility of erroneous inference results, provides a correct means of defining any anomaly from a dynamic point of view towards the fuzzy KBS. By imposing a constraint on the results of inference for an anomaly not to occur, one creates definitions of the anomalies, that can be verified by involving the inference process, and thereby the fuzzy inference operator in the analysis. This was formerly identified as the core idea behind the static line of thought in fuzzy verification literature. Summarized, this way of obtaining a valid anomaly checking procedure boils down to, taking into account the requirements of principle P1, taking as starting point the idea behind principle P2, in order to eventually, if feasible, end up with a realization of principle P3.

5.3 Uncoupling Verification and Inference

It seems as though, in the light of the rationale underlying previous section, the ultimate goal in verification research is balled into the idea of attaining principle P3. This is essentially true, but one should nevertheless be careful in striving for what leads in the end to a *de facto uncoupling* of verification and inference. This uncoupling of verification from inference has to be seen as an a posteriori finding. Throughout any performed anomaly analysis one needs to explicitly take into account the dynamics of the fuzzy KBS, as clarified in the previous discussion. The fact that the results of this type of analysis are formulated in terms of necessary and/or sufficient conditions on the relationship between the sets of the fuzzy KB, often creates the illusion that inference is omitted from verification. Proof of the fact that temptation is great to succumb to this illusion, is given by an evaluation of current publications in verification literature (cf. section 4). Starting from the idea that the introduction of fuzzy set theory boils down to the transposition of classical anomaly definitions into fuzzy equivalents by merely introducing some sort of affinity, matching or similarity measure, can in the process of conceiving a verification theory be seen as a noble initiative and is by no means prohibited. Things, however, go wrong when in the quest for a good affinity, matching or similarity measure, one neglects to take into account the effect of the implication operator. In that sense, it also should not come as a surprise that results of a verification analysis can differ substantially according to the considered implication function.

6. Topics of Further Research

It seems useful to formulate some remarks in the direction of future research initiatives as to verification in fuzzy rule bases.

1. It looks feasible to keep on pursuing research in the line of thought of a dynamic analysis of anomalies. All but an exhaustive analysis of all possible rule base scenario's has been done up till now. The bulk of current contributions can be found in Dubois, Prade & Ughetto [2].

2. Research has to be conducted for anomalies that have not been addressed explicitly in literature yet : deficiency and circularity.

3. The process of reflecting on the input proposed by Yager and Larsen [1] enables the knowledge engineer to identify the zone within the input domain to a fuzzy rule based system where a problem of consistency could occur. How could this knowledge be used usefully in any consistency checking procedure ? We are thinking in terms of using this result in combination with other verification approaches proposed in literature, e.g. neural network learning and verification.

4. Is it theoretically feasible to introduce a concept of 'degree of anomaly' ? As an example we state the concept of degree of inconsistency $\alpha \in [0..1]$ proposed by Yager & Larsen [1]. To what extent does this conflict with current realizations in verification literature on fuzzy rule bases ? In this context, we directly think of the concept of normality of fuzzy sets, which forms the basis of current consistency research.

These considerations only form a very limited subset of potential research topics. They nevertheless are indicative of the bulk of work that still lies in front in the field of verification of fuzzy rule based systems.

7. Conclusion

In this paper, we identified and exemplified dual lines of thought, static and dynamic, underlying the construction of the in literature proposed verification models that try to extend the verification of classical rule bases to the case of fuzzy knowledge modeling, without needing a set of representative input. The major outcome of the confrontation between both approaches is that their results, stated in terms of necessary and/or sufficient conditions for anomaly detection starting from the same basic assumptions within a particular situation are difficult to reconcile. The duality framework introduced in section 4.1, enables us to put things in the right perspective. The analysis presented points out that the foundations underlying both views on verification proposed in literature are not incompatible with one another. At the origin of the observed duality in realizations of both rationale lies an error in the conception of the in literature proposed static approaches towards verification of fuzzy rule bases. Things essentially go wrong when in the quest for a good affinity, matching or similarity measure, one neglects to take into account the effect of the implication operator.

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